Analysis of the Traveling Performance of Planetary Rovers with Wheels Equipped with Lugs over Loose Soil

M. Sutoh¹, K. Nagatani², and K. Yoshida³

^{1,2,3} Department of Aerospace Engineering, Graduate School of Engineering, Tohoku University, Aoba 6-6-01, Sendai, Miyagi 980-8579, JAPAN; Ph +81-22-795-6993; email:{sutoh, keiji, yoshida}@astro.mech.tohoku.ac.jp

ABSTRACT

The surfaces of the Moon and Mars are covered with loose soil. In such conditions, planetary rovers can get stuck and even cause failure of an exploration mission. To avoid such problems, the wheels of planetary rovers typically have lugs (i.e., grousers) on their surface, which substantially improve their traveling performance. However, there exists no theoretical method to determine a suitable lug interval. In this study, we first modeled the linear traveling speed of a wheel with lugs and provided guidelines for determining a suitable lug interval, as well as the corresponding terramechanical stress models. Next, to verify the suitable interval for lugs, we performed traveling tests using a two-wheeled rover with wheels having different numbers of lugs of different heights. According to the experimental results, when a wheel has three lugs in a range where normal stress is generated beneath it, it can travel with constant speed while the traveling performance is substantially improved.

INTRODUCTION

Mobile robots (rovers) have played a significant role in NASA's Martian geological investigations. The use of rovers in missions significantly increases the area that can be explored, thereby improving the scientific return from the mission. However, the surfaces of the Moon and Mars are covered with loose soil, and numerous steep slopes are found along their crater rims. Under such conditions, wheeled rovers can get stuck and even cause mission failure. To avoid such problems, many research groups have studied the traveling performances of wheeled rovers on the basis of terramechanics (Iagnemma, 2004; Ishigami, 2007).

Conventionally, terramechanics has mainly been used to study large vehicles such as dump trucks (Bekker, 1960; Wong, 2001). Parallel fins called lugs (i.e., grousers) or convex patterns on the wheels for large vehicles have the advantage of increasing the traveling performance. However, the extent of the influence is not so large that there is no need for further increasing the lug effectiveness. On the other hand, it has been reported that lugs substantially improve the traveling performances of lightweight vehicles such as planetary rovers (Bauer, 2005; Liu, 2008; Sutoh, 2011). Therefore, it is important to have a better understanding of the effect of lugs on the traveling performance of planetary rovers.

Some reports have investigated the influence of lugs on the traveling performance of lightweight wheeled rovers. Ding et al. reported experiments to evaluate the influence of lug height, lug spacing, and lug inclination angle on the traveling performance of a wheel (Ding, 2010). In addition to the experimental approach, a method to estimate the traveling performance of wheels equipped with lugs, which uses the discrete element method (DEM), has been proposed by Nakashima et al. (Nakashima, 2010). Furthermore, a dynamic model for small lightweight vehicles with wheels having lugs was recently introduced by Irani et al. (Irani, 2011). These studies have provided a better understanding of the effect of lugs. However, from the point of view of designing wheels, there is no theoretical method to determine a suitable lug interval.

Our objective is to provide guidelines for determining the lug interval on a wheel. To this end, we first model the linear traveling speed of a wheel. Next, we discuss the suitable lug interval on a wheel, as well as the relationship between the traveling speed and the normal stress distribution beneath the wheel. Finally, to verify the proposed suitable lug interval, we perform traveling tests using a two-wheeled rover with wheels having different numbers of lugs of different heights.

We first explain the linear traveling speed model of a wheel with lugs. Next, the relationship between the traveling speed and the normal stress distribution is derived and discussed, along with the terramechanical stress distribution models. The above experiments and their results are reported in detail.

LINEAR TRAVELING SPEED MODEL OF WHEEL WITH LUGS

For a planetary rover to travel over a slope covered with loose soil, a force is required to pull its weight. This force is called the drawbar pull. The drawbar pull is defined as the difference between the thrust developed by the rover's wheels and the wheels' motion resistance. Generally, a wheel with lugs obtains its thrust from shear stress developed by the surface of the wheel and lugs.

Over a steep slope, rover wheels require significant drawbar pull and the surface of the wheel may not be able to generate enough thrust to move the rover forward. Therefore, we assumed that only when the lugs develop thrust, the wheels acquire enough drawbar pull for the rover to move forward. Thus, the rover's linear traveling speed periodically changes in cycle corresponding to lug interval. Under such conditions, small lug intervals, i.e., large numbers of lugs, contribute to shorten the cycle. Furthermore, we considered that the peak values of the linear speed would be the same for different lug intervals.

Based on the above description, for wheels with different lug intervals, the relationship between the rotation angle of the wheel and the linear speed of the wheel over a steep slope is obtained, as shown in Figure 1. In the figure, α denotes an angle where the linear speed changes, i.e., where lugs develop thrust, and β denotes an angle between lugs (see Figure 2).

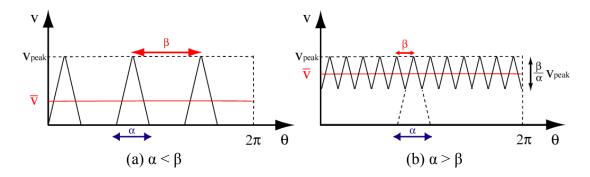


Figure 1. Linear traveling speed of wheels with lugs; α and β denote an angle where lugs develop thrust and an angle between lugs, respectively.

When β is greater than α , we express the linear speed ν of the wheel, as (see Figure 1 (a))

$$v = \begin{cases} \frac{2v_{peak}}{\alpha} (\theta - n\beta) & \left(n\beta \le \theta \le n\beta + \frac{\alpha}{2} \right) \\ -\frac{2v_{peak}}{\alpha} (\theta - n\beta) + 2v_{peak} & \left(n\beta + \frac{\alpha}{2} \le \theta \le n\beta + \alpha \right) & (n = 0, 1, 2, \dots, N - 1) \\ 0 & (otherwise) \end{cases}$$
 (1)

where N is the number of lugs on a wheel, and v_{peak} is the peak speed of the wheel. Further, the mean linear speed \bar{v} is calculated as

$$\frac{1}{v} = \frac{\frac{1}{2}\alpha v_{peak} \times \frac{2\pi}{\beta}}{2\pi} = \frac{1}{2}\frac{\alpha}{\beta} v_{peak}.$$
(2)

On the other hand, when β is smaller than α , we express ν as (see Figure 1(b))

$$v = \begin{cases} \frac{2v_{peak}}{\alpha} (\theta - n\beta) + \left(1 - \frac{\beta}{\alpha}\right) v_{peak} & \left(n\beta \le \theta \le n\beta + \frac{\beta}{2}\right) \\ -\frac{2v_{peak}}{\alpha} (\theta - n\beta) + \left(1 + \frac{\beta}{\alpha}\right) v_{peak} & \left(n\beta + \frac{\beta}{2} \le \theta \le n\beta + \beta\right) \end{cases}$$
 $(n = 0, 1, 2,, N - 1)$ (3)

The mean linear speed v is calculated as

$$\bar{v} = \frac{2\pi v_{peak} - \frac{1}{2} \frac{\beta}{\alpha} v_{peak} \beta \times \frac{2\pi}{\beta}}{2\pi} = v_{peak} \left(1 - \frac{1}{2} \frac{\beta}{\alpha} \right). \tag{4}$$

Using Equations (2) and (4), the relationship between the lug intervals and the mean linear speed is obtained, as shown in Table 1. According to Table 1, when β is greater than $\alpha/3$, shorting the lug interval substantially contributes to improving the mean

Table 1. Relationship between lug intervals and mean linear speed.

β/α	2	1	1/2	1/3	1/4	1/5	1/6
- v/V _{peak}	0.25	0.5	0.75	0.83	0.88	0.9	0.92

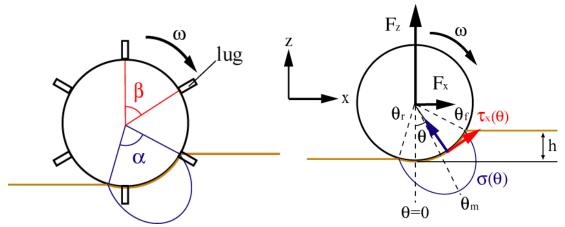


Figure 2. Angle where lugs develop thrust, α , and angle between lugs, β .

Figure 3. Normal stress σ and shear stress τ_x generated beneath wheel.

speed. On the other hand, when β is smaller than $\alpha/3$, it only slightly improves the mean speed. When β is $\alpha/3$, the mean speed is already 83% of the peak speed. From this finding, we concluded that if the lugs are equipped with an interval of $\alpha/3$ on the surface of the wheel, the wheel can travel with a constant speed and the traveling performance of the wheel will be substantially improved.

ESTIMATION OF NORMAL STRESS DISTRIBUTION BENEATH WHEEL

As mentioned above, α , the angle where lugs develop thrust, is a key parameter to determine a suitable lug interval. We considered α as the range where normal stress is generated beneath the wheel. Therefore, α can be estimated on the basis of the terramechanical normal stress distribution models. In this section, we introduce the terramechanical stress distribution models and discuss α and the suitable lug interval for a wheel.

Normal stress distribution model. When a wheel travels over loose soil, a normal stress σ and a shear stress τ_x , are generated beneath it (see Figure 3). We assume that the normal stress distribution is roughly the same for a wheel with or without lugs. Therefore, by estimating σ and α for a wheel without lugs, we can determine the suitable lug interval for the wheel.

On the basis of Reece's model, the normal stress σ is determined from the following equation (Ishigami, 2007):

$$\sigma(\theta) = \sigma_{\max} \left[\cos \theta - \cos \theta_f \right]^n \qquad \left(\theta_m \le \theta \le \theta_f \right)$$

$$\sigma(\theta) = \sigma_{\max} \left[\cos \left\{ \theta_f - \frac{\theta - \theta_r}{\theta_m - \theta_r} \left(\theta_f - \theta_m \right) \right\} - \cos \theta_f \right]^n \qquad \left(\theta_r \le \theta \le \theta_m \right)$$
(5)

where σ_{max} denotes the maximum stress and is determined from the following equation (Reece, 1965):

$$\sigma_{\text{max}} = \left(ck_c + \rho k_{\phi}b\right)\left(\frac{r}{b}\right)^n \tag{6}$$

where b and r denote the wheel width and wheel radius; k_c and k_{φ} denote the pressuresinkage moduli; n denotes the sinkage exponent, which is an inherent parameter of the soil; and c and ρ are the cohesion stress of the soil and the soil bulk density, respectively.

In Equation (5), θ_f and θ_r denote the entry angle and departure angle of the wheel, respectively (see Figure 3). The entry angle θ_f is defined as the angle from the vertical to the point at which the wheel initially makes contact with the soil and is expressed as

$$\theta_f = \cos^{-1}(1 - h/r) \tag{7}$$

where h is the wheel sinkage. The departure angle θ_r is defined as the angle from the vertical to where the wheel departs from the soil and the value is generally assumed to be $\theta_r \approx 0$. Further, θ_m is the specific wheel angle where the normal stress is maximum and

$$\theta_m = (a_o + a_1 s)\theta_f \tag{8}$$

where a_0 and a_1 are parameters that depend on the wheel-soil interaction. Their values are generally assumed to be $a_0 \approx 0.4$ and $0 \le a_1 \le 0.3$ (Wong, 1967).

Shear stress distribution model. The expression for the shear stress τ_x beneath a wheel was formulated by Janosi and Hanamoto (Janosi and Hanamoto, 1961), as follows:

$$\tau_x = (c + \sigma(\theta) \tan \phi) \left[1 - e^{-j_x(\theta)/k_x} \right]$$
 (9)

where c denotes the cohesion stress of the soil, φ is the internal friction angle of the soil, and k_x is the shear deformation modulus, which depends on the shape of the wheel surface. Further, j_x , which is the soil deformation, can be formulated as a function of the wheel angle θ :

$$j_x(\theta) = r \left[\theta_f - \theta - (1 - s) \left(\sin \theta_f - \sin \theta \right) \right]$$
 (10)

where s denotes a slip ratio and is defined as (Wong, 2011):

$$s = \frac{v_d - v}{v} = 1 - \frac{v}{v_d}. (11)$$

Here, v and v_d denote the actual linear speed of a vehicle and the angular speed of the wheel, respectively.

Vertical force. To prevent a wheel from sinking into loose soil, a vertical force is required. The vertical force F_z is obtained by integrating the vertical components of σ and τ_x from the entry angle θ_f to the departure angle θ_r as follows (Wong, 2011):

$$F_z = rb \int_{\theta_r}^{\theta_f} \{ \tau_x(\theta) \sin \theta + \sigma(\theta) \cos \theta \} d\theta$$
 (12)

where b and r denote the wheel width and wheel radius, respectively.

The vertical force F_z satisfies the following equation:

$$F_{z} = Mg \tag{13}$$

where M is the normal load of the wheel, and g is the gravitational acceleration.

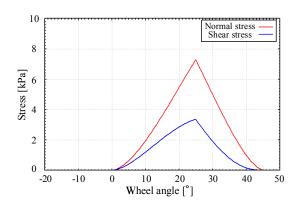


Figure 4. Normal and shear stress distribution beneath a wheel without lugs, which has a diameter of 150 mm and a width of 100 mm (normal load of the wheel was 2.0 kg).

Solving for F_z from Equations (5), (9), (12), and (13) leads to an estimate for the normal and shear stress distribution beneath a wheel.

Estimation for \alpha. As mentioned in the previous section, we assumed that the rover moves forward over a steep slope, only when the lugs develop thrust. That is, when a wheel without lugs travels over such a slope, the slip ratio is close to 1.0. Therefore, to determine a suitable lug interval, α when the slip ratio of a wheel without lugs is 1.0 is important.

For a wheel without lugs, which has a diameter of 150 mm and a width of 100 mm, to travel over Toyoura standard sand (JIS R 5200), the normal and shear stresses beneath the wheel are obtained, as shown in Figure 4. Here, the normal load of the wheel was set to 2.0 kg, and the soil parameters of Toyoura standard sand are listed in Table 2 (Yoshida, 2004). According to this figure, in case the slip ratio s is 1.0, we obtained an α of 45°. Thus, to substantially improve the traveling performance of the wheel, the wheel should be equipped with lugs with an interval of 15° (α /3). In other words, the wheel should be equipped with 24 lugs with identical spacing intervals on its surface.

Table 2. Soil parameters and values.

parameter	value	unit	
С	0.0	[kPa]	
φ	38.0	[°]	
ρ	1.49	$[g/cm^3]$	
k_x	0.03	[m]	
k_c	0.0	$[N/m^{n+1}]$	
k_{arphi}	1.20×10^3	$[N/m^{n+1}]$	
n	1.70	[-]	
a_0	0.4	[-]	
a_1	0.15	[-]	

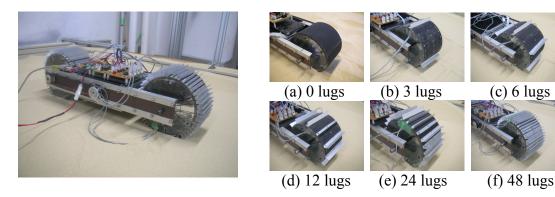


Figure 5. Two-wheeled rover.

Figure 6. Wheels equipped with different numbers of lugs.

VERIFICATION EXPERIMENT

To verify the abovementioned suitable lug interval on the wheel, we performed traveling tests using a two-wheeled rover with wheels with different numbers of lugs of different heights. In the tests, the rover traveled over a slope, and we measured the linear traveling speed of the rover. In this section, the experiments and experimental results are reported in detail.

Two-wheeled rover. We developed a lightweight two-wheeled rover with interchangeable wheels (see Figure 5). The distance between the front and rear wheels of the rover was 400 mm, and the rover weight was set to 4.0 kg for different wheel types. Motion measurement systems with optical sensors and laser sources (Nagai, 2010) were mounted onto the rover to measure the actual rover's traveling speed, without external devices embedded in the target environment.

Eleven wheel types were developed. Except for the wheel without lugs, the other wheels have two different lug heights and five different numbers of lugs, as shown in Figure 6. To match the experimental conditions to those presented in the previous section, the wheel has a diameter of 150 mm and a width of 100 mm; each lug has a height of 5 or 15 mm; that is, the wheel has a diameter of 160 or 180 mm including the lug heights. The wheel surfaces were covered with sandpaper to simulate the interaction between soil particles.

Experimental overview and conditions. The two-wheeled rover, with the abovementioned eleven types of wheels, was used to perform traveling tests in a sandbox. The sandbox has a length, width, and depth of 1.5 m, 0.3 m, and 0.15 m, respectively, and was filled with Toyoura standard sand. The sandbox can be manually inclined to change its slope angle. In the experiments, the slope angle was fixed at 16° (see Figure 7). The angular speed of the wheel was fixed at 2.50 rpm, and we measured the linear traveling speed of the rover along with the rotation angle of the wheel. Each trial was conducted under identical soil conditions, and three trials were conducted for each condition.



Figure 7. Two-wheeled rover travels over a slope.

Experimental results and discussion. To verify the suitable lug interval presented in section 3, we plotted the relationship between the rotation angle of the wheel and the linear speed of the rover for cases with a fixed lug height (see Figures 8 and 9). According to the figure, it was observed that in cases of wheels with 3, 6, and 12 lugs, the speed of the rover periodically changes in cycles corresponding to the lug interval. Meanwhile, in cases of wheels with 24 and 48 lugs, the speed is almost constant. In both cases, the peak values of the speed are almost the same for different lug intervals. In other words, an increase in the number of lugs contributes to shorten the cycles in the linear speed, but not to change their peak value. These experimental results correspond to our assumption.

According to Figures 8(a) and 9(a), in the case of wheels with 3 lugs, it was observed that α , a measure of the angle where the linear speed changes, was about $40^{\circ}\sim50^{\circ}$. This is almost the same angle as that estimated from the normal stress model. This means that, as we assumed in section 3, lugs develop thrust in a range where normal stress is generated beneath the wheel. Furthermore, the angle was almost the same for different lug heights. From this, we concluded that α was determined from the diameter, width, and the normal load of the wheel, regardless of lug heights.

According to the above discussion, we concluded that if the lugs are equipped with lug intervals of $\alpha/3$ on the surface of the wheel, the wheel can travel with constant speed and the traveling performance of the wheel is substantially improved.

According to Figures 8(b) and 9(b), for both the lug height cases, the value of the traveling speed of the wheel with 24 lugs is slightly higher than that of a wheel with 48 lugs. This result does not correspond to Equation (4). Furthermore, if an angle between lugs β is close to 0, i.e., the number of lugs is infinite, the wheel has a larger diameter. However, it has been reported that wheels with lugs have a higher traveling performance than those with a large diameter (Sutoh, 2011). Further study is therefore needed on the application of Equation (4) to wheels with a large numbers of lugs.

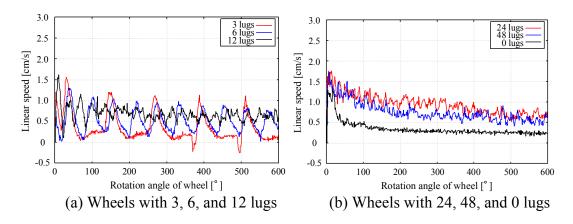


Figure 8. Rotation angle of wheel versus linear traveling speed (for 5mm-height lug).

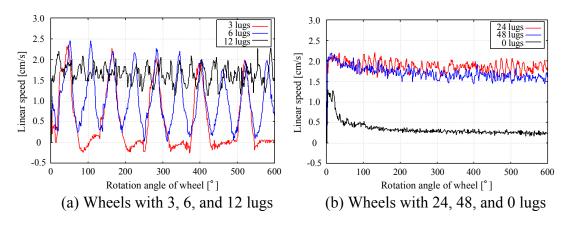


Figure 9. Rotation angle of wheel versus linear traveling speed (for 15mm-height lug).

CONCLUSION AND FUTURE WORK

In this study, the linear traveling speed of a wheel with lugs was modeled. Next, the suitable lug interval on a wheel is discussed on the basis of the normal stress distribution beneath the wheel. To verify the suitable lug interval, we performed traveling tests using a two-wheeled rover with wheels having different numbers of lugs with different heights. According to the experimental results, when a wheel has three lugs in a range where normal stress is generated beneath it, it can travel with constant speed and its traveling performance is substantially improved.

Further study is necessary on the application of the proposed model to wheels with large numbers of lugs.

REFERENCES

Bauer, R., Leung, W., and Barfoot, T. (2005). "Experimental and simulation results of wheel-soil interaction for planetary rovers." *Proceedings of 2005 IEEE/RSJ*

- International Conference on Intelligent Robots and Systems, pp.586-591, Edmonton Alberta, Canada
- Bekker, M.G. (1960). *Off the road locomotion*. Ann Arbor, MI: The University of Michigan Press.
- Ding, L., Gao, H., Deng, Z., Nagatani, K., and Yoshida, K. (2010). "Experimental study and analysis on driving wheels' performance for planetary exploration rovers moving in deformable soil." *Journal of Terramechanics*, 48(1):27-45.
- Iagnemma, K., Kang, S., Shibly, H., and Dubowsky, S. (2004). "Online terrain parameter estimation for wheeled mobile robots with application to planetary rovers." *IEEE Transactions on Robotics*, 20(5):921-927.
- Irani, R.A, Bauer, R.J, Warkentin, A. (2011). "A dynamic terramechanic model for small lightweight vehicles with rigid wheels and grousers operating in sandy soil." *Journal of Terramechanics*, 48(4):307-318.
- Ishigami, G., Miwa, A., Nagatni, K., and Yoshida, K. (2007). "Terramechanics-based model for steering maneuver of planetary exploration rovers on loose soil." *Journal of Field Robotics*, 24(3):233-250.
- Janosi, Z. and Hanamoto, B. (1961). "The analytical determination of drawbar pull as a function of slip for tracked vehicle in deformable soils." *Proceeding of the 1st International Conference on Terrain-Vehicle Systems*, Torino, Italy.
- Liu, J., Gao, H., and Deng, Z. (2008). "Effect of straight grousers parameters on motion performance of small rigid wheel on loose sand." *Information Technology Journal*, 7(8):125-1132
- Nagai, I., Watanabe, K., Nagatani, K., and Yoshida. K. (2010). "Noncontact Position Estimation Device with Optical Sensor and Laser Sources for Mobile Robots Traversing Slippery Terrains." *Proceeding of IEEE/RSJ International Conference on Intelligent Robots and Systems*, pp.3422-3427, Taipei, Taiwan.
- Nakashima, H., Fujii, H., Oida, A., Momozu, M., Kanamori, H., Aoki, S., Yokoyama, T., Shimizu, H., Miyasaka, J., and Ohdoi, K. (2010). "Discrete element method analysis of single wheel performance for a small lunar rover on sloped terrain." *Journal of Terramechanics*, 47(5):307-321.
- Reece, A. R. (1965). "Principles of soil--vehicle mechanics." *Proceedings of the Institution of Mechanical Engineers*, 180:45-66.
- Sutoh, M., Nagatani, K., and Yoshida, K. (2011). "Evaluation of influence of surface shape of wheel on traveling performance of planetary rovers over slope." *Proceedings of 17th ISTVS International Conference*, Blacksburg VA, USA.
- Wong, J. Y. and Reece, A. R. (1967). "Prediction of rigid wheel performance based on the analysis of soil-wheel stresses: Part I. Performance of driven rigid wheels." *Journal of Terramechanics*, 4(1):81-98.
- Wong, J.Y. (2001). *Theory of Ground Vehicles 3rd edition*. New York: John Wiley and Sons.
- Yoshida, K., Mizuno, N., Ishigami, G., and Miwa, A. (2004). "Terramechanics-Based analysis for slope climbing capability of a lunar/planetary rover." *Proceedings of 24th International Symposium on Space Technology and Science*, 2004-k-06, Miyazaki, Japan.